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Spatial game approach to describe risky agents in evacuation situations

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ABSTRACT OF MASTER'S THESIS

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Threat to survival launches a primitive fight-or-flight reaction both in animals and humans. Both individual actions and the actions of others affect an individual's survival when escaping as a part of a crowd. Human characteristics play a big role in decision making under evacuation circumstances. Attitudes towards risks make some people try their luck, and some others to act as carefully as possible.

An individual's view of the seriousness of a threat in the current situation can be modeled bu using a personal cost function. The shape of the cost function determines whether one is more risk-averse or risk-taking. This thesis seeks to find out how crowd egress flow is affected when evacuees' cost functions differ from each other. Previous studies have treated evacuees as homogenous individuals who all have the same cost function.

Evolutionary game theory serves as the decision making framework for this study. Classical Hawk-Dove game can be used to model human behavior alternatives, e.g., an individual to play Impatient or Patient under an evacuation situation. This individual's behavior can be observed by the other evacuees, who react to this behavior according to their own cost functions.

The study in this thesis is limited to two different types of evacuees: risk-averse and risk-taking. The model developed will reveal new kinds of phenomena that do not occur when all evacuees are considered homogenous. For example, mixing the two types of evacuees in the same crowd will cause a formation of a certain area in the middle of the crowd where all the risk-averse evacuees take the action Patient, and all the risk-taking evacuees take the action Impatient.

Keywords:	evacuation, cellular automaton, evolutionary game theory, risk attitude
Language:	English

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Uhka selviytymiselle laukaisee alkukantaisen taistele tai pakene -reaktion sekä eläimissä että ihmisissä. Sekä yksilölliset että muiden tekemät toimet vaikuttavat yksilön selviytymiseen paettaessa osana väkijoukkoa. Ihmisten luonteenpiirteillä on merkittävä osa päätöksenteossa evakuointitilanteissa. Asenteet riskejä kohtaan saavat toiset kokeilemaan onneaan ja toiset toimimaan niin varovaisesti kuin mahdollista.

Yksilön näkemystä uhan vakavuudesta nykyisessä tilanteessa voidaan mallintaa henkilökohtaisella kustannusfunktiolla. Kustannusfunktion muoto määrittää onko yksilö riskiä karttava vai riskihakuinen. Tämä diplomityö pyrkii selvittämään kuinka väkijoukon ulosvirtaukseen vaikuttaa evakuoitavien toisistaan poikkeavat kustannusfunktiot. Edelliset tutkimukset ovat ajatelleet evakuoitavia homogeenisina yksilöinä, joilla on kaikilla sama kustannusfunktio.

Tässä tutkimuksessa käytetään evoluutiopeliteoriaa pelaajien, tai agenttien, toiminnan ennustamiseen. Esimerkiksi klassisen Haukka-Kyyhky-pelin toimintavaihtoehdot ovat “Kärsimätön” ja “Kärsivällinen” evakuointitilanteissa. Muut havaitsevat yksilön käyttäytymisen ja reagoivat tähän oman kustannusfunktionsa mukaisesti.

Tässä diplomityössä tutkimus on rajoitettu kahdentyyppisiin evakuoitaviin: riskiä karttaviin ja riskihakuisiin. Kehitetyllä mallilla tehdyt simuloinnit tuottavat uudenlaisia ilmiöitä, joita ei tapahdu samantyyppisten agenttien tapauksessa. Esimerkiksi kahta eri tyyppiä olevien agenttien sekoittaminen samaan agenttijoukkoon muodostaa joukon keskelle tietyn alueen, jossa kaikki riskiä karttavat evakuoitavat käyttäytyvät kärsivällisesti ja kaikki riskihakuiset evakuoitavat käyttäytyvät kärsimättömästi.

Asiasanat:	evakuointi, soluautomaatti, evoluutiopeliteoria, riskiasenne
Kieli:	Englanti

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Ilmari Pärnänen

Abbreviations and acronyms

C	cost of conflict
CA	cellular automaton
D_{ij}	value of dynamic floor field in cell (i, j)
$E(I, J)$	payoff to individual adopting strategy I, when the opponent adopts strategy J
EGT	evolutionary game theory
ESS	evolutionary stable strategy
HD	Hawk-Dove game (also known as chicken game or snowdrift game)
k_D	sensitivity parameter associated with the dynamic floor field
k_S	sensitivity parameter associated with the static floor field
ξ_i	random force affecting agent i in social force model
PD	prisoner's dilemma game
S_{ij}	value of static floor field in cell (i, j)
T_{ASET}	available safe egress time
T_i	estimated evacuation time for agent i
$u(T_i; T_{ASET})$	cost function for agent i
V	value of resource, which is obtained after won contest
$W(I)$	average fitness of individuals that choose strategy I

Contents

Chapter 1

Introduction

People have died due to crowd disasters as long as there has been mass gatherings. To prevent this kind of incidents, the state authority and building designers have established safety requirements. Ignoring to make a sufficient safety plan may end up in a situation of numerous casualties - both deaths and injuries. A recent example of this kind of disaster was a stampede that broke out at Dussehra festival 2014 in Gandhi Maidan, India. Casualties included 33 people dead and more than 20 injured [20].

Especially in media, the term *panic* is commonly used to describe the state of mind of a crowd in a threatening situation. Panicking is considered to make people become irrational, selfish and obsessed to acquire short-term personal benefit. However, studies carried out by social psychologists have revealed that this kind of irrational behavior rarely occurs on individuals and whether the concept of panic should be even used at all as a technical term [22]. Thus, it is reasonable to assume that stampedes and similar crowd disasters take place under conditions where humans behave *rationally.*

This thesis focuses on pedestrian behavior in evacuation situations. Two influential agent-based computational models are presented in Chapter 2. The first one is social force model, which describes pedestrian motion in continuous space as the resultant of physical and socio-psychological forces. The other one is cellular automaton model, in which agents movement direction depends on floor fields. These floor fields depend on the geometry of the space the agents are evacuating themselves from, and the recent history of other agents' movement and location in the space. Both social force and cellular automaton models act as a core to several developed evacuation simulation software products [17].

Considering evacuation simulations, it is naturally important to compare computational results to observed human behavior in evacuation situations, for example under the conditions of a disaster. Chapter 3 begins with introducing a game theory related topic: *evolutionary game theory*. Evolutionary games were originally developed for analyzing the competition, reproduction and dynamics of biological lifeforms and to describe the strategic interaction of them. To add *decision making abilities* to the evacuees in a computational model, game theory is coupled with evacuation models. The evacuees in the model have two strategies to choose from: Impatient or Patient. The strategy choice alters the evacuees behavior and depends on physical conditions, such as smoke and fire, and the other evacuees actions. Simulation results with these coupled models can be used to understand mechanisms behind crowd disasters. The game that evacuees play with each other is *spatial* in nature because in a big crowd an individual is able to interact only with the people in his local surroundings. The theory of spatial games is described in Chapter 3, too.

So far, computational evacuation models have treated evacuating agents as identical decision-makers. This means that two agents behave exactly the same way in a similar situation. Different persons in an identical situation may find completely opposite alternatives as the optimal way of acting. In the context of evacuations, many human characteristics have potential influence in the modeling of decision making: bravery, caution, determination and so on. To be able to take into account the effect of these characteristics, new features must be added to the current models.

This thesis seeks to find a way to enhance the evacuee's decision making, by making a classification of agent types. The different types will represent agents, that have the same strategy set but different *cost functions*. This way, the optimal strategy in a situation may not be anymore unique, but depend on agent's type. The different cost functions reflect the *risk attitude* of the agent types. This allows certain proportions of the evacuating crowd being more risk-averse or risk-taking than the rest. This extension to the evacuation model is presented in Chapter 4.

The analysis of the new evacuation model features continues in Section 5, where the effect of multiple utility functions is studied further through simulation. Both static equilibrium configurations of agents in front of the exit, and behavior of moving agents in a cellular automaton are simulated. After that, the results are analyzed. Finally, in Chapter 6 potential future research is discussed.

Chapter 2

Evacuation models

Pedestrian behavior can be modeled in several ways. While this thesis aims to improve the decision making abilities of computational agents, at start a review of some different significant modeling frameworks must be done. Section 2.1 discusses social force model developed by Helbing [8]. In the model agents move in continuous space and time, taking into account different force affecting the agents. Some of these forces are actual physical forces while the others are socio-psychological phenomena that can be expressed like forces. An alternative to Helbing's continuous framework is cellular automaton model developed by Schadschneider [23] in section 2.2. In this model agents move in discrete time steps in a discrete grid. The evacuees move in the grid according to transition probabilities, which depend on the static and dynamic floor fields. These fields are derived from the geometry of the room and the agents' past movement in the grid.

2.1 Social force model

At the beginning there are totally n evacuees trying to get out from a space. Each agent i has: a mass of m_i, a desired walking speed v_i^0 and a desired direction to move to $\mathbf{e}_i^0$. The actual velocity of an agent is $\mathbf{v}_i$ and the agent has a characteristic acceleration time τ_i, which describes how long it takes for the agent to accelerate from the actual velocity to the desired velocity. In addition the agent attempts to avoid getting too close to obstacles on its way, namely other agents j and walls W. This tendency can be expressed as part of "interaction forces" $\mathbf{f}_{ij}$ and $\mathbf{f}_{iW}$. Now, as the model assumptions are set, the motion of the agent can be written as an equation:

$$m_i \frac{d\mathbf{v}_i}{dt} = m_i \frac{v_i^0 \mathbf{e}_i^0 - \mathbf{v}_i}{\tau_i} + \sum_{j(\neq i)} \mathbf{f}_{ij} + \sum_W \mathbf{f}_{iW} + \xi_\mathbf{i}, \tag{2.1}$$

where $\xi_\mathbf{i}$ is a small personal random force. The random force must be included in the model, because without it the agents might jam in a head-on encounter with equal oppositely directed total moving forces. The position of the agent $\mathbf{r}_i$ can then be obtained from the velocity $\mathbf{v}_i = d\mathbf{r}_i/dt$.

The force $\mathbf{f}_{ij}$ repelling the agents from one another consists of three parts:

$$\mathbf{f}_{ij} = \mathbf{f}_{ij}^{\text{social}} + \mathbf{f}_{ij}^{\text{body}} + \mathbf{f}_{ij}^{\text{slid}}. \tag{2.2}$$

Social force $\mathbf{f}_{ij}^{\text{social}}$ is the psychological part of this interaction force. It can be written as

$$\mathbf{f}_{ij}^{\text{social}} = A_i \exp[(R_{ij} - d_{ij})/B_i]\mathbf{n}_{ij}, \tag{2.3}$$

where A_i and B_i are constants describing the strength and range of the social force, $R_{ij} - d_{ij} = R_i + R_j - \|\mathbf{r}_i - \mathbf{r}_j\|$ is the sum of agents i and j 2D projection radii, defined as in Figure 2.1, subtracted by the distance between agents' centers, and $\mathbf{n}_{ij} = (\mathbf{r}_i - \mathbf{r}_j)/d_{ij}$ is a normalized vector pointing from agent j to agent i. As in [13], each agent is considered to form of three spheres, the big one describing the main body and the small ones the arms.

In the model, two agents are defined to touch each other if $R_{ij} - d_{ij} \geq 0$. While it might at first seem odd this subtraction to get positive values, in Figure 2.1 there is presented a case when it happens. After the agents touch, the two other components of the interaction force activate. The 'body force' and 'sliding friction force' can be written as

$$\mathbf{f}_{ij}^{\text{body}} = kg(R_{ij} - d_{ij})\mathbf{n}_{ij} \tag{2.4}$$
$$\mathbf{f}_{ij}^{\text{slid}} = \kappa g(R_{ij} - d_{ij})\Delta v_{ji}^t \mathbf{t}_{ij}, \tag{2.5}$$

where k and κ are large constants, $\mathbf{t}_{ij} = (-n_{ij}^2, n_{ij}^1)$ is the tangential direction when agents i and j are close to each other, and $\Delta v_{ji}^t = (\mathbf{v}_j - \mathbf{v}_i) \cdot \mathbf{t}_{ij}$ is the tangential velocity difference of the agents. The function $g(x)$ is added to make these forces active only if the agents touch; $g(x) = x$, for $R_{ij} - d_{ij} \geq 0$, otherwise $g(x) = 0$. As can be seen, unlike $\mathbf{f}_{ij}^{\text{social}}$ these two forces are actual physical forces.

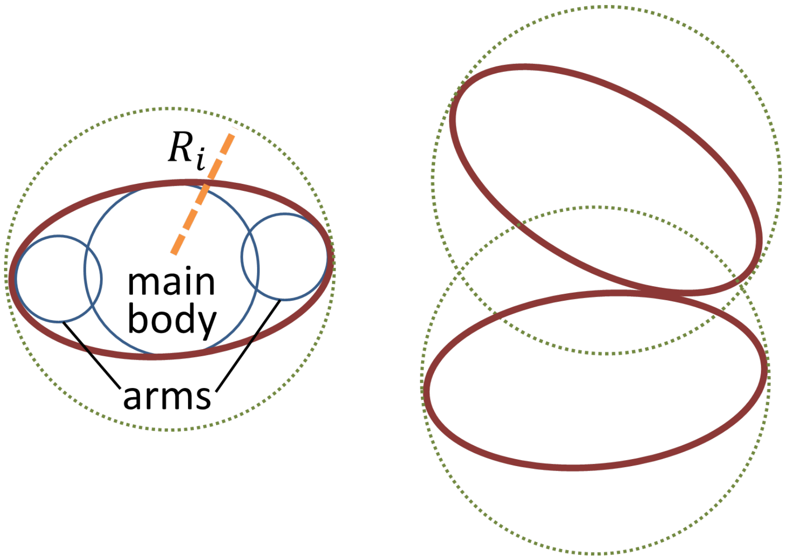


Figure 2.1: On the left-hand side there is presented a 2D projecton of an agent modeling a pedestrian. On the right-hand side there is presented a case, where two agents end up really close to each other. In this case body and sliding friction forces begin to influence the agents.

Agent i's interaction with walls W is treated analogously as interaction with another agent j. Hence, repulsive force towards a wall can be written as

$$\begin{aligned}\mathbf{f}_{iW} = \{A_i \exp[(R_i - d_{iW})/B_i] + kg(R_i - d_{iW})\}\mathbf{n}_{ij} \\ -\kappa g(R_i - d_{iW})(\mathbf{v}_i \cdot \mathbf{t}_{iW})\mathbf{t}_{iW}.\end{aligned} \tag{2.6}$$

The social force model presented here is just a good starting point to represent the crowd dynamics, and it can be modified with additional features. For example, to Equation (2.3) a term can be added that scales the quantity of the social force $\mathbf{f}_{ij}^{\text{social}}$ so that pedestrians in front of agent i have larger repulsive impact than agents behind agent i at the same distance [9]. Also, some attractive social forces can be added to simulate people's tendency to gravitate towards a familiarity group like family members or friends, or social forces can be made time-dependent [7]. Simulations done with Helbing's social force model have been able to describe many observed large crowd evacuation phenomena. For example, it gives a physical interpretation on why in a narrow straight hallway pedestrians with the same desired walking direction $\mathbf{e}^0$ tend to move in a line formation, or why through two small doors next to each other there is larger people flow than through a large door which is broader than the two small doors in total [9].

In my research work in evacuation modeling at Aalto University Systems Analysis Laboratory I've used social force model in simulations done with FDS+Evac software developed by Technical Research Center of Finland (VTT). Original FDS (Fire Dynamics Simulator) is a platform for simulating effects of fire in buildings and FDS+Evac is a evacuation simulation module coupled with the platform [16]. Even though social force model based FDS+Evac is in many ways an excellent software for modeling pedestrians' evacuation behavior, making simulations showed also a couple of major drawbacks of the approach. One drawback is the low computational efficiency of the model. Continuous space and time with very large number of forces affecting each agent in a evacuating crowd take really long time for the results to be computed. Another drawback is that some observed evacuation phenomena, that have been verified with social force model, couldn't be replicated with FDS+Evac in certain special geometry. Counterflow simulation failures in a squared shaped space with a squared shaped obstacle in the center are examples of this. They are explained in the following subsection. Simulations were carried out in collaboration with Simo Hostikka's and Timo Korhonen's Fire safety technology team at VTT.

2.1.1 Counterflow simulations with FDS+Evac

Helbing showed in [11] that with a high people density the social forces make pedestrians tend to *form lanes* with other pedestrians that have the same desired walking direction $\mathbf{e}^0$. This phenomenon can also easily be seen everyday for example in shopping malls when there is a rush hour and people, who try to make their way to a cashier, walk in a line towards the cashier among other people walking randomly to all other directions in the rush. Even more interesting phenomenon was showed in [10], which relates to the strength of the random forces $\xi_\mathbf{i}$. The strength ξ, also called strength of noise, can be interpreted analogous to temperature if a dense group of pedestrians is thought as particles flowing in a system. Increasing the strength makes pedestrians move more randomly around their optimal trajectory to their preferred destination and this can be interpreted as the nervousness of the pedestrians. What is interesting is that actually a high ξ destroys the lane formation and ends up the pedestrians getting stuck in a "crystallized" structure, when usually increasing temperature makes particles to transit into a a "gaseous" state. Because of the analogy of the strength ξ and temperature, the observed effect has been named *freezing by heating*.

In our first test setting for FDS+Evac the results of Helbing for lane formation and freezing by heating are studied. Agents are divided half and half in two types that have opposite desired walking directions in a narrow hallway. The initial distribution of the two kinds of agents is spread randomly in the hallway and then seen if the system reaches a stable or metastable state. Both ends of the hallway are also equipped with a feature called *periodic boundary conditions*, which means that if the agents move outside of either the left or right boarders, they will re-enter to the other side. As expected, with a low ξ lanes are formed and agents just continue to flow in a metastable state. If ξ is high, freezing by heating occurs and agents end up stuck in a stable state.

In the second test setting the geometry of the hallway is altered so that, instead of periodic boundary conditions, there is a square geometry with a squared shaped obstacle in the center. Desired walking directions are replaced correspondingly with a desired rotation direction with relation to the center of the area. Instead of similar results compared to setting one, lane formation does not appear but agents end up in a freezing by heating configuration with all values of ξ. This example shows, that with certain test settings FDS+Evac fails to produce an expected phenomenon. This failure is not actually the fault of the software, but the social force model has too simple rules on dynamics and further improvents in the theory must be done to fix these kind of errors. The results of the two test settings just described are illustrated in Figure 2.2.

2.2 Cellular automaton model

The evacuation space is divided into a grid of cells which are identical in size. Each cell has two possible states: empty or occupied. The state of the cell describes whether there is an agent located in the cell or not, and the size of the cell is defined so that not more than one agent can occupy a single cell. The state of a cell is updated each discrete time step t with an update rule that arises from the states of the cells in its *neighborhood* at time $t-1$. For example, an agent can't move occupying a cell closer to the exit in its neighborhood if the cell is already occupied. The agents' movement towards the exit can be observed by looking at the change in occupied cells in different times in the grid. This kind of system is called a cellular automaton (CA).

At start, the size of the cells and length of time steps must be chosen. Based on empirical evidence of an average pedestrian size and walking speed, a cell in the CA grid is chosen to be 40 x 40 cm and time step 0.3 s long

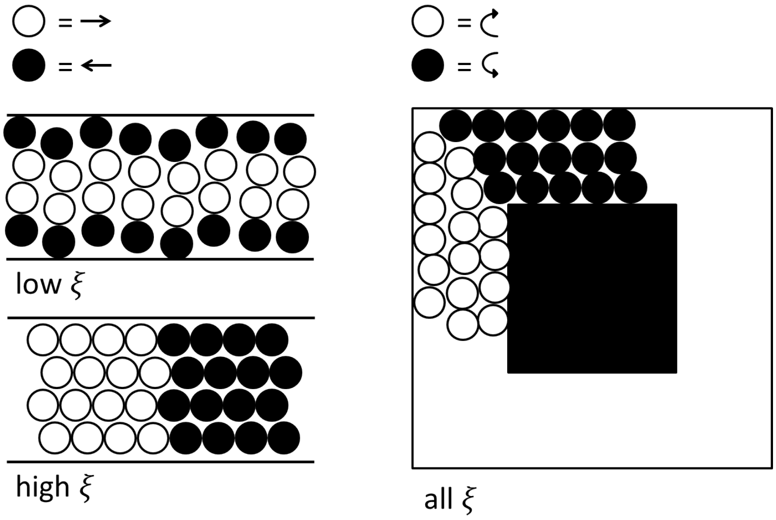
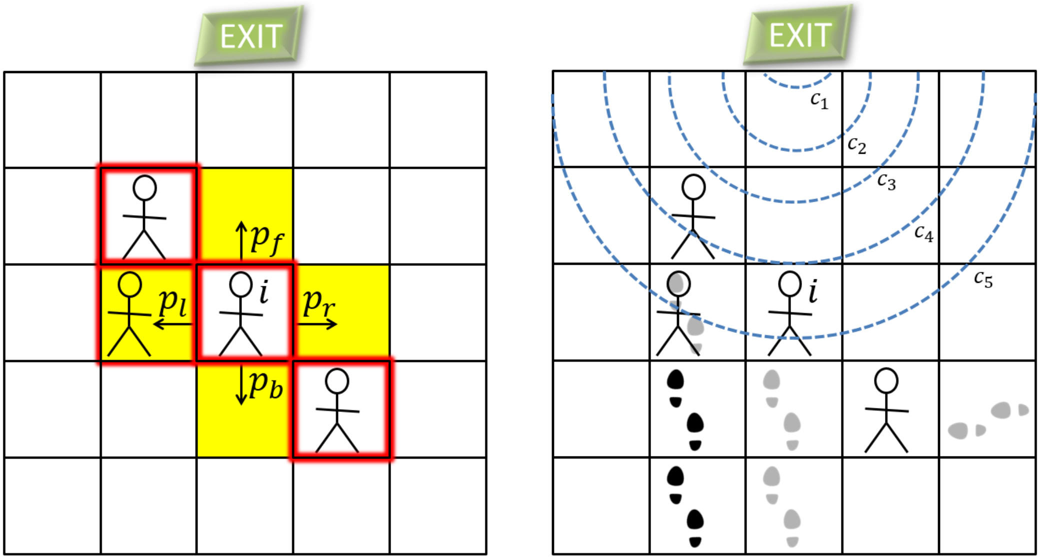


Figure 2.2: Illustration of the counterflow simulations. Original measurements were carried out with FDS+Evac software in summer 2012. In the pictures on the left-hand side the hallways have periodic boundary conditions. In the picture on the right-hand side the black square in the center describes an obstacle through which the agents cannot move.

[23]. An agent can move to an empty cell in its von Neumann neighborhood, which contains the four cells located orthogonally round the cell the agent is currently occupying (forward, back, left and right). Each of the cells in an agent's neighborhood contain a probability p to transit into that cell. To define these probabilities, the concept of *floor fields* affecting the cells must be introduced. In a way, floor fields reflect the intelligence of pedestrians and indicate an attractive direction to proceed. The first floor field is called static floor field S, which reflects the geometry of the room. The value S_{ij} is strongest at the cell in front of the exit and the field strength in a cell decreases the further away the cell is located from the exit. Values of static floor field don't vary across time.

The other attractors besides the location of the exit are *virtual traces* left by the other evacuees. The dynamic floor filed D is defined so that at the beginning of the evacuation it has the value zero in each cell and every time an agent moves out of a cell (i, j), the value of the dynamic floor field in that cell increases $D_{ij} \rightarrow D_{ij} + 1$. The values of D diminish over time and the process resembles diffusion and decay of a bosonic field in statistical physics. Pedestrians moving in the grid kind of "drop a boson" ($+1$ in D_{ij}) into the cell when moving out from that cell. Over time, with certain probabilities α



(a) Von Neumann neighborhood.

(b) Floor fields.

Figure 2.3: An instance of an evacuation situation in Schadschneider's CA model. In picture (a), agent i's von Neumann neighborhood at time step t is colored in the picture with yellow. Occupied cells that can't be entered are marked with a person icon and rounded with red color. In picture (b), blue constant value contours ($c_1 > c_2 > c_3 > \ldots$) of static floor field S describe the attractiveness of the exit. Foot steps indicate the virtual trace that forms the dynamic floor field D. Black foot steps indicate a cell from which already two agents have moved out, hence D is stronger in those cells than in cells with gray foot steps. If agent i is facing towards the exit, the highest probability to move is p_f in the front cell. The cell on the left is occupied, and thus $p_l = 0$ and i couldn't even move there.

and δ, the boson can either diffuse to one of its neighboring cells (changing the place of the +1) or decay completely away, respectively [15]. The basic principles of the CA model described so far are illustrated in Figure 2.3.

Now, when all the necessary concepts are introduced, our next task is to define the transition probabilities p_{ij}. Weighting parameters k_S and k_D in Equation (2.7) below, mark the importance of fields S and D. Parameters n_{ij} and ξ_{ij} are needed to point out occupied cells and obstacle cells (for example walls) in the grid. Now, the transition probabilities can be written as

$$p_{ij} = N e^{k_S S_{ij}} e^{k_D D_{ij}} (1 - n_{ij}) \xi_{ij} \tag{2.7}$$
$$N = \left[\sum_{(i,j)} e^{k_S S_{ij}} e^{k_D D_{ij}} (1 - n_{ij}) \xi_{ij} \right]^{-1},$$

where n_{ij} is 1 for occupied cells and 0 for empty cells, and ξ_{ij} is 1 for normal cells and 0 for obstacle cells. So, at each time step t, all agents can either move in their von Neumann neighborhood or stay in their current cell, and Equation (2.7) points out probabilities to each of these up to five alternative actions. This simultaneous update rule for all cells is also called parallel update scheme. Simultaneous updating also exposes the agents to *conflict situations*, in which more than one agent attempts to occupy the same empty cell. In case of a conflict, the model chooses randomly an agent involved to the conflict to proceed and occupy the empty cell, hence preventing the other agents from moving during that time step.

By altering the weighting parameters k_S and k_D, different kinds of behaviors can be simulated with a CA model. If the weight for floor field S is given a high weight compared to the weight for floor field D, the static attractor, i.e., the exit, dominates the dynamic virtual traces and the agents attempt to only use cells giving them route straight towards the exit. This kind of behavior is called *ordered regime*. On the other hand, if the weight on floor field D is much greater than the weight on the floor field S, agents just follow the virtual traces that the others have left to the grid. Over time, agents end up in groups ("herds") following each other and not necessarily follow routes towards the exit. The herding behavior is called *disordered regime*. Between these two behaviors there is also a third alternative, which occurs with suitable values of k_S and k_D. The intermediate behavior is called *cooperative regime*, and agents behaving in this way are attracted both from the location of the exit and also from cells through which there is a high level virtual trace left by the other agents. Cooperative agents will not try to advance through the shortest route to the exit like ordered agents if there is nearby another longer route that has greater values in the dynamic field because many other agents have just recently gone through that route. A plot of parameter values of k_S and k_D with corresponding regime domains is presented in Figure 2.4.

Cellular automaton model has much better computational efficiency than social force model presented in section 2.1. Space and time are discrete, and agents interact only with their local neighborhood. The local nature of CA helps when treating more complex evacuation geometries. If in social

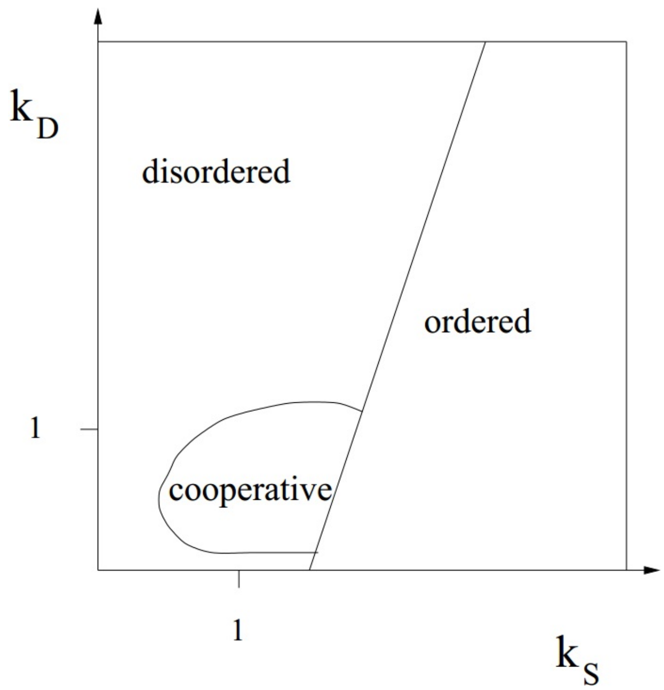


Figure 2.4: Different behavior regimes as function of parameters k_S and k_D, as depicted in [14].

force model two agents are really close to each other but there is a wall between them, then naturally there should not be a social force interaction between the two agents. If the model doesn't check if there are visual blocks like walls between the agents, simulation results may contain errors (see an example of a social force software failure in section 2.1.1). In general, the computational effort in social force models increases proportionally to the square of the number of agents in the simulation, while in cellular automaton the effort is proportional to the size of the grid. [2] The main drawback of CA compared to social force model is that because of the local nature of agent interactions the effect of a bigger crowd pushing can't be seen in CA. For example, looking at the agent movement couple of meters from the exit door: in CA this movement is indifferent whether there is no one or lots of people coming behind the agents near the door. In real life deaths caused by crowd stampedes happen usually when a big crowd pressure is fatal to a single individual and this can be simulated with social force model.

Chapter 3

Game theoretical background

In order to build up an agent-based evacuation model, where the agents are equipped with simple behavioral rules, certain game theory related topics are introduced in this chapter. First one is evolutionary game theory (EGT) in Section 3.1. Although, originally introduced for purposes to model evolution of biological lifeforms, many of the results of classical game theory can also be obtained from evolutionary framework. Most important concepts of EGT, such as Hawk-Dove game, Evolutionary Stable Strategies and Bishop-Cannings theorem, are covered. Spatial games are introduced in Section 3.2. As argued in the previous chapter, the strongest interactions between evacuating pedestrians happen within their local neighborhood, thus theory of spatial games is suitable for the purpose of model enhancement. An example of game theoretical decision making abilities coupled with an evacuation model is reviewed shortly in section 3.3.

3.1 Evolutionary games

The presentation in this section follows closely the text of Maynard Smith[18]. In an evolutionary game, strategies are behavioral phenotypes. A certain phenotype specifies what an individual will do in any situation it finds itself from. The utility the individual contestants try to maximize is called *fitness*. The higher the fitness, the more the individual is able to produce offspring resembling itself to the next generation.

Hawk-Dove game (HD) is defined as follows. Two animals have a contest over a *resource* that has total value V, which is the amount that individual's

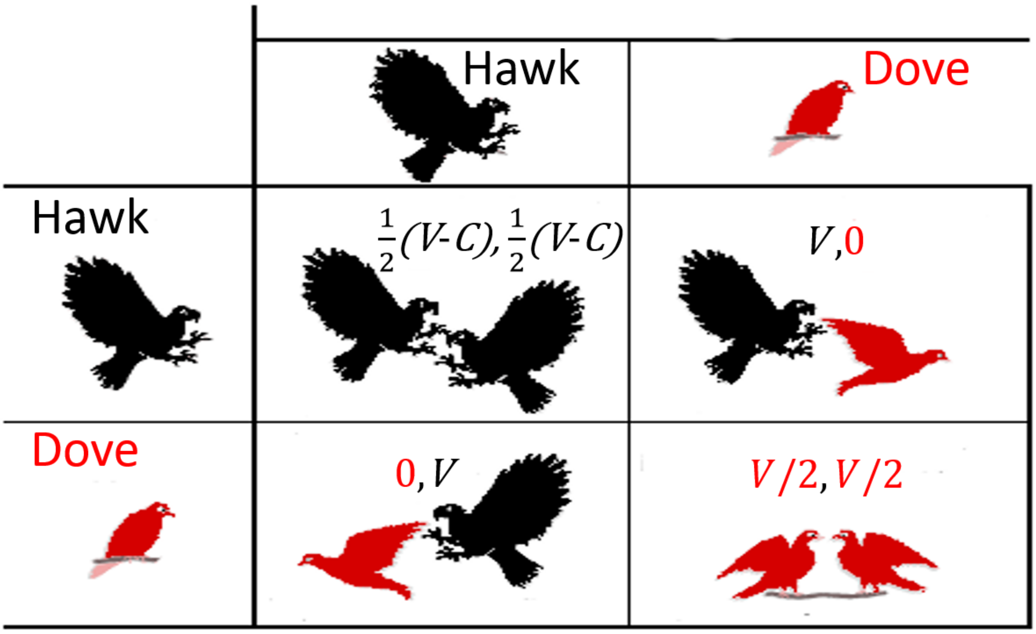


Figure 3.1: Payoff matrix for the Hawk-Dove game.

fitness will increase if the resource is obtained fully. The resource could be, for example, a territory that is favorable for breeding. The loser of the game would need to go to a less favorable area to breed, and the more poor conditions would lead to a smaller offspring of the loser. If breeding in the favorable area would produce on average 5 offspring to the individual, and breeding in the less favorable area would produce 3 offspring, then the amount of V that is obtained would be $5 - 3 = 2$. The game is played so, that first both individuals choose to adopt one out of two strategies available, and then behave or act according to the strategy chosen. The two strategies are:

- *Hawk* (H): escalate and fight for the resource until injured or the opponent retreats;
- *Dove* (D): never escalate, retreat immediately if the opponent escalates.

So, by escalating the animal tries to force the opponent to retreat so that it could obtain the whole resource for itself. However, if the opponent escalates as well, the animals end up fighting. At some point of the fight, one of the animals will be injured and forced to retreat, while the other obtains the resource V. In addition to retreating, the loser of the fight will suffer a cost C reducing its fitness because of being injured.

The payoff matrix of HD is presented in Figure 3.1. Single cell of the matrix represents the outcome to the contestants of that particular choice of strategies. The first value in the cell is the outcome for the row contestant (who plays Hawk, the upper row, or Dove, the lower row) and the second value is the outcome for the column contestant (in the similar way). The values in the matrix can be explained by looking at the different kinds of encounters possible in a Hawk-Dove game.

i *Hawk vs. Hawk*: Both contestants have 50% chance of obtaining the resource V or suffering the cost C from injury. Thus, the outcome is presented as the expected value of these two alternatives.

ii *Hawk vs. Dove*: Hawk escalates and obtains the resource V. Dove retreats without being injured and the change in its fitness is 0.

iii *Dove vs. Dove*: Because neither escalates, the resource V is shared equally by the two contestants.

Next step is to extend the population of individuals from two animals to a larger group. Consider an example: a large population, in which all animals adopt either H or D as their strategy, and then the animals paired off randomly. Let

- W_0: starting fitness of all individuals,
- p: frequency of animals that choose H strategy,
- $W(I)$: average fitness of individuals that choose strategy I,
- $E(I, J)$: payoff to individual adopting strategy I, when the opponent adopts strategy J.

After the whole population has gone through a single pairwise contest, the average fitness of the two different strategists in the group are

$$\begin{aligned} W(H) &= W_0 + pE(H,H) + (1-p)E(H,D), \\ W(D) &= W_0 + pE(D,H) + (1-p)E(D,D). \end{aligned} \tag{3.1}$$

For simplicity, the animals are considered to reproduce their kind asexually. As stated before, the number of offspring is proportional to fitness. In the next generation, the frequency of Hawks p' will be

$$\begin{aligned} p' &= pW(H)/\bar{W}, \\ \bar{W} &= pW(H) + (1-p)W(D), \end{aligned} \tag{3.2}$$

where $\bar{W}$ is the average fitness of all individuals in the game.

Using Equation (3.2) repeatedly gives the changes of the frequencies of the different strategies over time. This dynamics is a special case of *replicator dynamics*, which describes in general how successfully certain strategy spreads in a population with n different strategies. Like in all dynamical systems, the interesting question is under which conditions the system reaches a stable state. The stability criteria are now derived for a general case, and it can then be easily applied to HD.

Let I be a stable strategy. In other words, I has the property that, if I is adopted by almost all of the members in the population, then the fitness of this kind of typical member adopting the stable strategy is greater than any possible mutant adopting a different strategy J. If I wasn't a stable strategy, the mutant strategy J would invade the population after rounds of replicator dynamics, because the mutant strategy would lead to greater fitness over time than I. Keeping this in mind, recalling Equation (3.1) and setting p to be a small frequency of mutant strategy J, while most of the population consist mainly from adopters of stable strategy I, the average fitness of I and J strategists are:

$$\begin{aligned} W(I) &= W_0 + (1-p)E(I,I) + pE(I,J), \\ W(J) &= W_0 + (1-p)E(J,I) + pE(J,J). \end{aligned} \tag{3.3}$$

Strategy I is stable, thus by definition $W(I) > W(J)$. Since p is small, but arbitrary, we get for all $J \neq I$, that

$$\text{either} \qquad E(I,I) > E(J,I) \tag{3.4a}$$

$$\text{or} \qquad E(I,I) = E(J,I) \quad \text{and} \quad E(I,J) > E(J,J). \tag{3.4b}$$

If strategy I satisfies one of these *standard conditions* (Equation (3.4)), then I is an **evolutionary stable strategy** (ESS). However, it should be remembered that standard conditions apply only to case with infinite population, asexual inheritance and pairwise contests.

Next, we apply this to find all evolutionary stable strategies to HD. The strategy D is not an ESS, because $E(D,D) < E(H,D)$ (neither of conditons in Equation (3.4) are satisfied), and thus a population consisting mainly from Doves could be invaded by the mutant (Hawk). Strategy H may be an ESS iff $\frac{1}{2}(V-C) > 0 \Leftrightarrow V > C$. That is, if the resource has greater absolute value than the cost from injury, it is worth always to escalate.

But what is the ESS in case $V < C$? Clearly then a population consisting purely from either D or H can't maintain a stable state if a mutant tries to invade it. First, a case is studied in which an individual can change its strategy between H and D over time. Later, it is answered whether there is a stable state of a population of animals mixed out of fixed Hawk and Dove individuals.

Let I be a strategy: "play H with probability P, else play D". In this case, the offspring of parent I is neither H nor D, but the next generation will resemble the parent in such a way that the offspring has the same probability P of playing H (and else playing D) as its parent. This kind of strategy, in which the individual chooses randomly with certain probabilities one from the set of "pure", non-stochastic strategies, is called a *mixed strategy*. So, the question is: is there a value of P, which makes mixed strategy I an ESS? To be able to answer, the following theorem is needed.

The Bishop-Cannings theorem. *If I is a mixed ESS with support** $a, b, c \ldots$*, then* $E(a, I) = E(b, I) = \ldots = E(I, I)$.

Proof. By contradiction. Suppose an element a in the support of the ESS strategy I has the property

$$E(a, I) < E(I, I).$$

Strategy I can be expressed in the form $Pa + (1 - P)X$, where X is either pure or mixed strategy adopted by I when it does *not* act as a. Then

$$\begin{aligned} E(I, I) =& PE(a, I) + (1 - P)E(X, I) \\ <& PE(I, I) + (1 - P)E(X, I) \\ \Leftrightarrow E(I, I) <& E(X, I). \end{aligned}$$

This can't be true, because I is an ESS. With the same argument, also $E(a, I) \not> E(I, I)$. Thus, it must be

$$E(a, I) = E(I, I).$$

Since a was chosen arbitrarily from the support of I, the same holds also for all the other elements in the support. $\square$

* The support of I is a set of all pure strategies that are played with a non-zero probability under I.

Now, if there is a probability P, which makes a mixed strategy I an ESS for Hawk-Dove game, it can be found by solving the Bishop-Cannings equation

$$E(H,I) = E(D,I). \tag{3.5}$$

In other words, the payoffs from playing either of the pure strategies H or D against mixed strategy I must be equal. We get

$$PE(H,H) + (1-P)E(H,D) = PE(D,H) + (1-P)E(D,D), \tag{3.6}$$

that is equivalent to,

$$\begin{aligned} \frac{1}{2}(V-C)P + V(1-P) &= \frac{1}{2}V(1-P), \\ P &= V/C. \end{aligned} \tag{3.7}$$

This result can also be generalized. For any similar two player two strategy payoff matrix

$$\begin{array}{ccc} & & \text{Player 2} \\ & & \begin{array}{cc} \text{I} & \text{J} \end{array} \\ \text{Player 1} & \begin{array}{c} \text{I} \\ \text{J} \end{array} & \begin{array}{|c|c|} \hline a,a & b,c \\ \hline c,b & d,d \\ \hline \end{array} \end{array},$$

if $a < c$ and $d < b$, then the probability P for mixed ESS strategy I is

$$P = \frac{b-d}{b+c-a-d}. \tag{3.8}$$

It has now been shown that $E(H,I) = E(D,I) = E(I,I)$. Equation (3.4b) still requires also that $E(I,D) > E(D,D)$, and $E(I,H) > E(H,H)$, must hold: we have

$$\begin{aligned} E(I,D) &= PV + \frac{1}{2}(1-P)V = (1+P)\frac{V}{2} > \frac{V}{2}, \\ E(I,H) &= \frac{1}{2}P(V-C) > \frac{1}{2}(V-C), \text{ since } V < C. \end{aligned}$$

Thus, it is shown that mixed strategy I with probability $P = V/C$ for playing H is evolutionary stable for HD. The interpretation for this is that, in HD, if the cost from injury is higher than the reward of acquiring the whole resource from a victorious fight, then it is expected that the players choose to play a mixed strategy.

What about the case in which the animals are not able to play mixed strategies and can only choose to be either pure Hawks or pure Doves? With condition $V < C$ only a mixed ESS could be found. Added to this, there might also be a *stable genetic polymorphism*; i.e., a mixture of pure Hawks and Doves in which the relative portions of H and D strategies stay constant over generations.

Let a population consist of pure H strategists with a frequency p and of pure D strategists with a frequency $1 - p$. If this kind of population reaches an equilibrium, then in that state the fitnesses $W(H)$ and $W(D)$ must be equal. So, we get

$$pE(H,H) + (1-p)E(H,D) = pE(D,H) + (1-p)E(D,D). \tag{3.9}$$

This equation is exactly the same as Equation (3.6), except that in the place of a mixed strategy probability P to play strategy H there is an equilibrium frequency p of fixed H strategists. Thus, the result in Equation (3.7) can also be used in the case of pure strategies, and over time a stable genetic polymorphism is always reached with frequency $p = V/C$.

Nevertheless, there are cases when this conclusion, the connection between mixed ESS and stable genetic polymorphism just found, doesn't anymore hold. If the amount of pure strategies is more than two, then in some games there might exist a mixed ESS with the corresponding polymorphism being unstable. The mathematical details of this stability problem is discussed in detail in [18], and in this thesis not covered further otherwise but through the next example.

Rock-Scissors-Paper game (RSP) is one of the worlds best known children's game. This version, however, has a small payment parameter ε for draws in it, as presented in the matrix below.

		Player 2		
		R	S	P
	R	ε, ε	$1, -1$	$-1, 1$
Player 1	S	$-1, 1$	ε, ε	$1, -1$
	P	$1, -1$	$-1, 1$	ε, ε

Three cases for parameter ε are considered.

1. $\varepsilon < 0$: The draw causes players to pay a cost. A mixed ESS exists $I = \frac{1}{3}R + \frac{1}{3}S + \frac{1}{3}P$. However, the population consisting only of pure strategies never converges to genetic polymorphism. Instead, the frequencies oscillate around the attractor $(\frac{1}{3}R, \frac{1}{3}S, \frac{1}{3}P)$ but never get closer to it.

2. $\varepsilon = 0$: The classical version of the game. No ESS exists, since inequality in Equation (3.4b) does not hold, but $E(I, R) = E(R, R) = 0$ (same applies also for S and P). This kind of equilibrium, where the optimal mixed strategy against any pure strategy is not strictly better than any pure strategy against itself, but equally good, is a weaker kind of equilibrium called *neutrally stable strategy* (NSS).

3. $\varepsilon > 0$: The draw causes players to gain a profit. Any kind of stability does not exist, since there are no probabilities to fulfill Equation (3.4).

The dynamics of the different cases are illustrated in Figure 3.2.

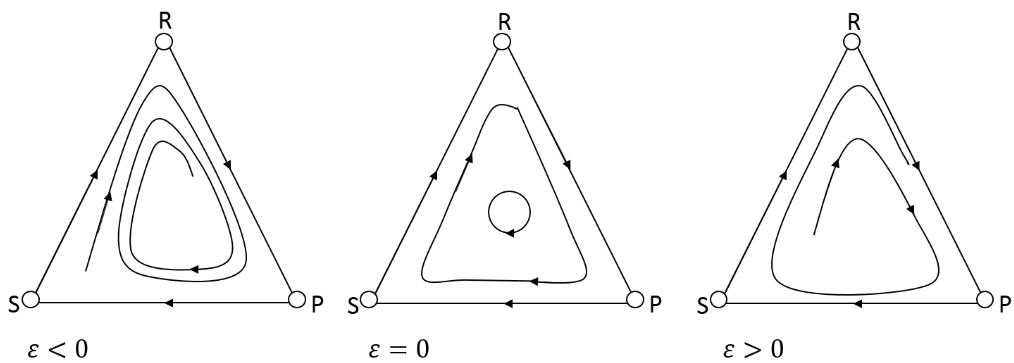


Figure 3.2: Replicator dynamics for RSP. [3]

3.2 Spatial games

In spatial game theory player population is distributed so that interactions only occur in a defined local neighborhood, instead of player decisions having a global effect on all other players [5]. In section 2.2, one this kind of neighborhood type was already presented, namely the von Neumann neighborhood, consisting of the orthogonal neighbors. Another widely used neighborhood type is Moore neighborhood, including both orthogonal and diagonal neighbors. In a regular square lattice, Moore neighborhood consists of the 8 neighboring squares round the one that is being observed.

The Hawk-Dove game, presented in the previous section, has been studied in a spatial framework [24]. In this version of HD, the agents are able to switch between H and D strategies and the interactions happen in their Moore neighborhood. As before, agents aim to choose a strategy that maximizes their payoff. However, this time the agents are only able to observe the strategies of their neighbors in the previous time step. The optimal strategy is chosen with an assumption that the neighbors will remain fixed with their strategies also during the next time step. Thus, agents are *myopic*, i.e. they have only a short-term memory lacking the knowledge of the events happened longer in the past, and they are also unable to predict the future.

Agent i chooses either strategy H or D and plays it simultaneously against its n neighbors. Different outcome alternatives are

$$W_i(H) = n_i^H E(H,H) + n_i^D E(H,D), \tag{3.10}$$
$$W_i(D) = n_i^H E(D,H) + n_i^D E(D,D), \tag{3.11}$$

where n_i^H and n_i^D are the amounts of Hawk and Dove neighbors, respectively. Recall Equation (3.7), which could also be interpreted so that, in a population of frequency V/C of Hawks, it is indifferent to choose between H and D strategies. Hence,

$$\frac{n_i^H}{n} < \frac{V}{C}, \qquad \text{choosing } H \text{ is profitable,} \tag{3.12}$$
$$\frac{n_i^H}{n} > \frac{V}{C}, \qquad \text{choosing } D \text{ is profitable,} \tag{3.13}$$
$$\frac{n_i^H}{n} = \frac{V}{C}, \qquad \text{the choice is indifferent.} \tag{3.14}$$

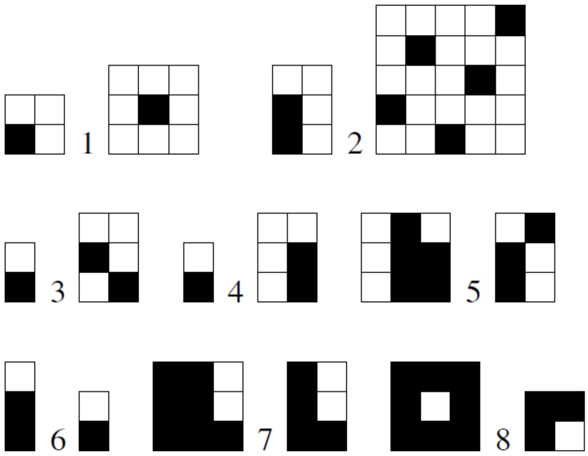


Figure 3.3: Elementary configuration blocks of spatial Hawk-Dove game with different values of fraction V/C. A black cell denotes a Hawk while a white cell denotes a Dove. With lowest values of V/C it is only optimal to play Hawk is all the surrounding agents are Doves, which corresponds to filling the lattic with number 1 blocks. Respectively, highest values of V/C corresponds filling the lattice with number 8 blocks (Dove only if all surroundig are Hawks), and the other cases between the two extremes. [24]

This way, each agent chooses the strategy to play according to what the current status of the neighborhood is compared to the fraction V/C, i.e., how big is the "temptation" to change the strategy. Adopting the decision rules in Equations (3.12)-(3.14) is analogous to adopting the ESS strategy in a normal HD. To avoid looping effects, strategy changing rate is "regulated". If agent's strategy from the previous iteration is the profitable one in the next iteration, the agent will hold on to its current strategy. If changing the strategy would be profitable, the agent will make the change with probability p, and hold on to the current strategy with probability $1 - p$.

First notable study of spatial game fractal patterns was made by Nowak in [19]. This classic study focused to a close relative of the Hawk-Dove game, namely the *prisoner's dilemma game* (PD), which is probably the most well-known example of game theory in the world. See Appendix for more information about PD. Studying the fractal patterns was also a major part of [24], where it was shown that, playing the spatial Hawk-Dove game with different values of fraction V/C, the equilibrium consists always of certain elementary configuration blocks (see Figure 3.3). In a static spatial game, equilibrium is the state in which the agents don't anymore change their strategies. Updating the strategies is done according to the *shuffle update rule*, which means that in the beginning of each time step the updating order of agents is first randomized, and then each agent updates its strategy one by one according to that order.

3.3 Spatial game for egress congestion

This subsection is based on [12] by Heliövaara et al. The model described here is also currently in use in the decision making module of FDS+Evac software described in section 2.1.

Let there be an evacuation situation with n agents. To each agent i, it is given a parameter T_i that denotes the estimated evacuation time of the agent. T_i can be written as

$$T_i = \frac{\lambda_i}{\beta}, \tag{3.15}$$

where λ_i is the number of agents between agent i and the exit, and β is the flow through the exit. The behavior of the agents depend on the values of a *cost function* $u(T_i; T_{ASET})$, where the *available safe egress time* T_{ASET} is a global parameter describing the maximum time the agents have to exit before the conditions in the occupied space become lethal. In addition to T_{ASET}, the shape of the cost function depends on conditions

$$u'(T_i) \geq 0, \quad u''(T_i) \geq 0. \tag{3.16}$$

In other words, inequalities in (3.16) means that $u(T_i)$ is increasing and convex.

The agents play a spatial game in their neighborhood. The game has two strategy alternatives: *Impatient* and *Patient*. Impatient agents attempt to reach the exit by pushing their neighbors, while patient agents attempt to avoid being in a physical contact with their neighbors. For interaction between neighboring agents i and j, a variable $T_{ij} = (T_i + T_j)/2$ is defined to describe their average estimated evacuation time. There are three possible interactions:

(a) *Impatien vs. Patient*: On this encounter the patient agent steps aside from its position to make way for the pushing impatient agent. The cost function of the impatient agent is decreased with amount

$$\Delta u(T_{ij}) = u(T_{ij}) - u(T_{ij} - \Delta T) \simeq u'(T_{ij})\Delta T \tag{3.17}$$

and the cost function of the patient agent is increased with the same amount, correspondingly.

(b) *Patien vs. Patient*: Here both agents try to avoid too close contact, so they keep their current positions and the cost functions remain unchanged.

(c) *Impatien vs. Impatient*: The two agents both push each other and end up in a conflict situation. The conflict exposes the agents to a risk of being injured, and this risk is described with an increase C, i.e. *cost of conflict*, in the cost functions.

By normalizing the change $\Delta u(T_{ij})$ in case (a) to 1, the following game matrix can be written:

$$\begin{array}{ccc|c|c|} & & \multicolumn{2}{c}{\text{Player 2}} \\ & & \text{Impatient} & \text{Patient} \\ \cline{3-4} \text{Player 1} & \text{Impatient} & \frac{C}{\Delta u(T_{ij})}, \frac{C}{\Delta u(T_{ij})} & -1, 1 \\ \cline{3-4} & \text{Patient} & 1, -1 & 0, 0 \\ \cline{3-4} \end{array} .$$

This game is either

$$\begin{array}{l} \text{PD, if } 0 < \dfrac{C}{\Delta u(T_{ij})} \leq 1, \text{ or} \\ \text{HD, if } \dfrac{C}{\Delta u(T_{ij})} > 1. \end{array} \tag{3.18}$$

So, depending on T_{ij}, the agents are playing in one of these two *game areas* defined by inequalities in (3.18). The result is familiar from section 3.1: it is always worth to rush if the potential decrease in cost function is greater than the cost from a conflict situation. This is also the ESS of PD, playing Impatient (Defect) in all situations. Otherwise, the strategy is chosen with

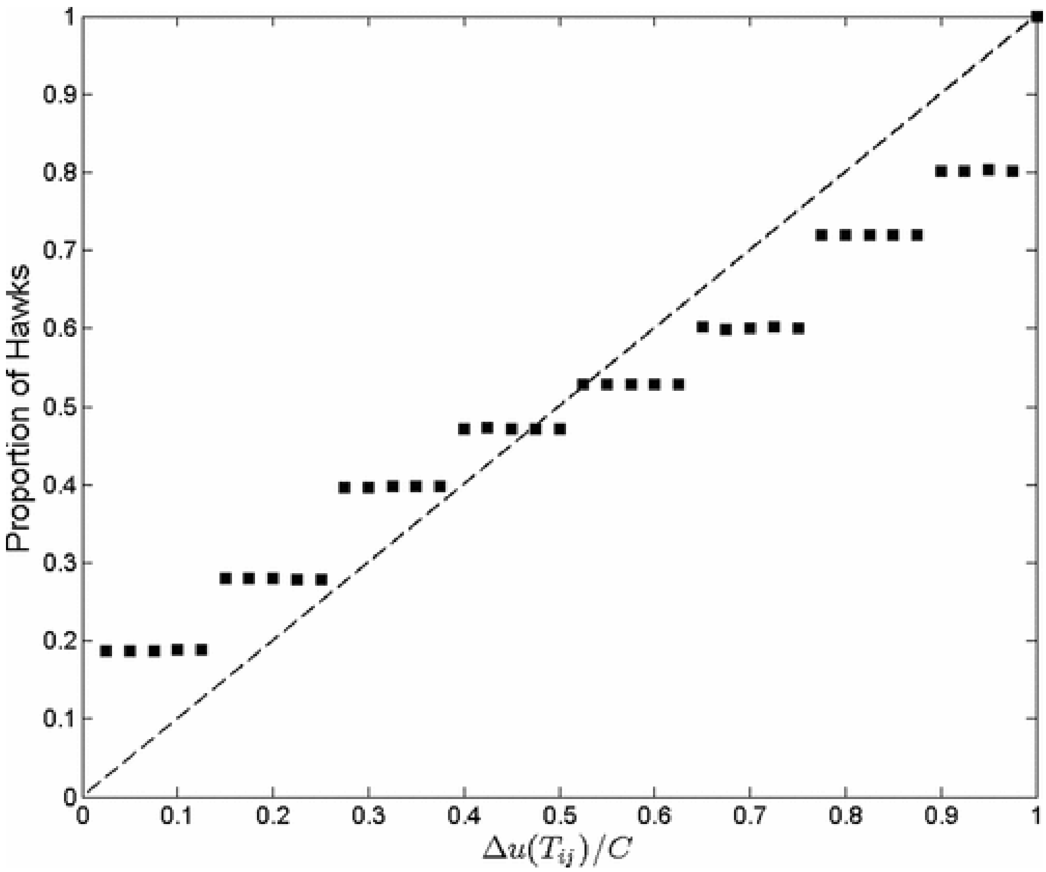


Figure 3.4: Proportion of Hawks (impatient agents) plotted against the parameter $\Delta u(T_{ij})/C$. The black squares indicate the fraction of Hawks in the spatial game equilibrium, and the dashed line the fraction of Hawks if the game would be played in a non-spatial well-mixed setting. [12]

mixed strategy probabilities out of Impatient (Hawk) and Patient (Dove). Actually, because the considered game is spatial, the mixed strategy alternative is transferred into action rules. These rules define the strategy choice of an agent based on the strategy structure of its neighborhood, like Equations (3.12)-(3.14) in section 3.2. Figure 3.4 shows that totally eight different equilibrium levels for the proportion of impatient agents exist as the function of parameter $C/\Delta u(T_{ij})$ (figure plotted for the inverse value $\Delta u(T_{ij})/C$.

The agents' strategies are updated over time according to each agent's best-response function BR_i. For agent i, the best-response strategy $s_i^{(t)}$ is defined by

$$s_i^{(t)} = BR_i(s_{-i}^{(t-1)}; T_i, T_{-i}) = \arg\min_{s_i' \in S} \sum_{j \in N_i} v_i(s_i', s_j^{(t-1)}; T_{ij}), \tag{3.19}$$

where N_i is the set of agent i's Moore neighbors, and $v_i(s_i', s_j^{(t-1)}; T_{ij})$ is the cost from game matrix agent i gets by playing s_i', when agent j has played $s_j^{(t-1)}$ at the previous time step. Also, here $s_{-i}^{(t-1)}$ denotes the strategies all other agents than i play at previous time step, and T_{-i} gives their estimated evacuation time. As in section 3.2, shuffle update scheme is used, and thus between time steps t and $t-1$ only one agent updates its strategy, according to the randomized order. The actual time evolves after a *simulation round* has gone by. A simulation round consists of n time steps, where n is the number of agents in the whole population.

Chapter 4

Risk attitude in evacuation situations

The spatial game for egress congestion presented in Section 3.3 depends on the agents' cost function. By altering the parameters of the cost function, different levels of threats can be simulated. However, in a real evacuation situation an individual doesn't have the information of the accurate available safe egress time, but the individual has to estimate this through observations. Thus, the parameters, the individual uses for information gathering and for decision making, contain uncertainty, and may differ among other individuals in the same situation.

It is difficult to accurately estimate the risks involved in an evacuation situation. Available information is processed through each individual's own social lenses that are constructed by the particular cultural context. [4] The effect of different kinds of *risk attitudes* in an evacuation situation has not yet been studied with simulation models. Hence, this thesis aims to find a way to implement different kinds of risk profiles to computational agents.

One way to model different risk attitudes is to introduce new strategies that correspond to these attitudes. An example of a three-strategy game, Rock-Scissors-Paper game, was introduced in section 3.1. As seen there, a game with more than two strategies may not have an ESS at all. In such particular non-ESS situations, the spatial strategy structure would alter over time even if the evacuating crowd wouldn't move closer towards the exit. Two-strategy game prevents this altering, because it has always an ESS. Fast convergence to an equilibrium, like ending up to ESS in two-strategy game, is important for the applicability of a simulation model. If the equilibrium can be found only with a few iterations that takes place under a simple updating scheme, it is conceivable that similar patters would occur also in real-life situations [12].

In this chapter, risk attitude implementation to crowd in an evacuation game is sought through introducing an idea that the shape of the cost function determines the agents risk attitude. So, the evacuating agents are let to have different cost functions when compared to each other. The different cost functions divide the agents in groups called *agent types*. The study is limited to cases with two different agent types in a same evacuation situation. This means that all the agents have the same strategies as before, Impatient or Patient, but some of them are more keen to play Impatient than others. The spatial game with two types of agents is coupled to the cellular automaton evacuation model. Results of the simulations done with the new model are presented in Chapter 5.

4.1 Multiple cost functions approach

As discussed in Section 3.3, the cost function of spatial egress congestion game depends on the average estimated evacuation time T_{ij} and available safe egress time T_{ASET}. Also, $u(T_{ij}; T_{ASET})$ is increasing and convex. If T_{ij} is small enough compared to T_{ASET}, the agents don't feel their situation threatening and don't try to optimize their exiting.

If $T_{ij} > T_{ASET}$, i.e. agents are really far away from the exit, the game the agents play becomes PD and all agents play only Impatient. As a counterpart, for small enough T_{ij} the agents don't play the egress game at all, which can happen really close to the exit. In other words, this kind of agents can be described as all playing only Patient. Parameter T_0 describes how short the time difference between T_{ij} and T_{ASET} must be to make the agents play the egress game. We choose

$$u(T_{ij}) = \begin{cases} 0, & \text{if } T_{ij} < T_{ASET} - T_0, \\ \frac{C}{2T_0}(T_{ij} - T_{ASET} + T_0)^2, & \text{if } T_{ij} > T_{ASET} - T_0. \end{cases} \tag{4.1}$$

This function satisfies the cost function requirements, because (i) game is not played when $T_{ij} < T_{ASET} - T_0$, (ii) after $T_{ij} > T_{ASET} - T_0$ function's quadratic increasing is convex, and (iii) $u'(T_{ASET}) = C$, which turns the HD to PD after $T_{ij} > T_{ASET}$.

Recall Equation (3.17) from the previous chapter. Now, the cost function gives

$$\Delta u(T_{ij}) \simeq u'(T_{ij})\Delta T = \frac{C}{T_0}(T_{ij} - T_{ASET} + T_0)\Delta T, \tag{4.2}$$

and by setting $\Delta T = 1$ for simplification, the defining parameter $C/\Delta u(T_{ij})$ of the spatial egress game is

$$\frac{C}{\Delta u(T_{ij})} \simeq \frac{T_0}{T_{ij} - T_{ASET} + T_0}. \tag{4.3}$$

Next, let k be the agent type. Parameters are now adjusted for the two different cost functions to reflect two different agent types under the same evacuating circumstances. A realistic assumption is to fix the difference $T^k_{ASET} - T^k_0$ to a constant value. In other words, even if there is multiple ways to evaluate the level of risk, the agents start to play the HD at the same time, which is at $T^k_{ASET} - T^k_0$. It is convenient to choose

$$T^k_{ASET} - T^k_0 = 0 \Leftrightarrow T^k_0 = T^k_{ASET}, \tag{4.4}$$

since the constant part of the piecewise Equation (4.1) can this way be removed. Equation (4.4) also means that all agents play the game, even those really close to the exit. In real-life, this could be interpreted as a situation, where all pedestrians want to evacuate as fast as possible.

An example of two cost functions is illustrated in Figure 4.1. Agents of type $k = 1$ use u^1 as their cost function and consider the conditions critical after $T^1_{ij} > 300$. Correspondingly, type $k = 2$ agents base their decision making upon cost function u^2 and start to play only Impatient after $T^2_{ij} > 100$. Note, that the evacuating conditions are the same for both agent types, and the different types are randomly distributed in the crowd without anyone knowing the type of any other agent but itself. The agent only observes the strategies of the agents in its Moore neighborhood, and can estimate its and their estimated evacuation time. This way, the agent types, and the corresponding cost functions, actually reflect personal risk attitude, rather than exact fact-based level of threat. Type 1 agents see the situation similar at $T^1_{ij} = 300$ as type 2 agents when $T^2_{ij} = 100$.

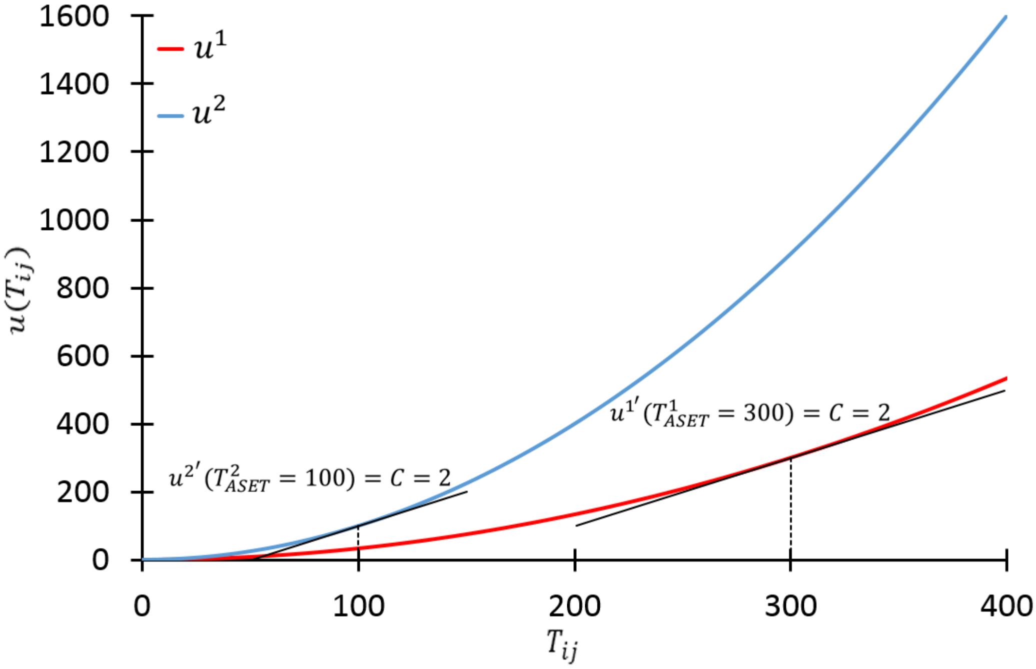


Figure 4.1: Illustration of two cost functions u^1 and u^2 with parameter values $T^1_{ASET} = 300$, $T^2_{ASET} = 100$ and $C = 2$.

4.2 Implementing different risk behaviors into spatial cellular automaton game

Von Schantz and Ehtamo coupled spatial game of egress congestion with cellular automaton simulation model in [26]. Originally, the parameters of CA are global in the model, and adjustments have to be done manually between simulations. In the coupled model, parameters of the simulation model are derived from the game the agents play, and can vary among the different agents in the same simulation.

Recall the different behaviors of CA agents listed in Section 2.2: ordered, disordered and cooperative. Ordered agents choose to follow the shortest path towards the exit, disordered agents follow each other in herds, and cooperative agents find following both shortest path and local flows attractive. If there would be only one agent evacuating, following the shortest path would naturally be the fastest way to evacuate. On the other hand, if in a case of many agents everyone would adapt this behavior, a jam would form in front of the exit. This is because if all agents try to move straight towards the exit, their paths will cross quite often, and that again causes conflict situations.

Because game theory always includes decision making among multiple agents, it is essential to discuss the treatment of encounters in CA. In Schadschneider's original version of the model presented in Section 2.2 *conflict situations* were resolved by random choice. This means that if more than one agent wants to enter a free cell at the same time, the model arbitrarily chooses one agent that is let to enter the desired cell, and the others have to stay at their current cells. In a later version of the model [14], a friction parameter μ was introduced to describe the probability that no agent in the conflict situation manages to occupy the desired free cell. The interpretation of this kind of friction is the hesitation the evacuees experience when ending up in a conflict situation, and this hesitating slows down the evacuation. Actually, friction parameter has been given also game theoretical interpretation, e.g. in [1, 6], where the parameter were thought to represent a situation of all agents playing Defect of PD.

The friction parameter μ has different effect on different behavioral regimes of CA. At this point, disordered regime is excluded from further considerations, because this behavior is thought to occur only in special cases of limited visual conditions, e.g., room filled with smoke. It was showed in [14] that the friction parameter had the strongest effect on evacuation times in the ordered regime. Without the friction parameter ordered regime evacuates the fastest, but after $\mu > 0.4$ ordered regime evacuates slower than the cooperative regime. This effect is called *faster-is-slower*, and can also be observed in simulations done with social force model presented in Section 2.1. Several agents trying to enter the shortest path cross more likely in each others way and end up in conflict situations, while a longer route with fewer agents on the way could be faster.

Now, spatial egress game strategies can be connected to CA's sensitivity parameters. Ordered regime corresponds to impatient behavior: agents try to move along the shortest path (static floor field) towards the exit and don't avoid ending up in conflict situations. Same way, cooperative regime corresponds to patient behavior: agents try to avoid conflict situations, so they prefer areas with higher local flows (dynamic floor field), where there is less probable to end up in conflicts. Remember, different regimes were produced purely out of sensitivity parameters k_S and k_D values. Thus, an agent is let to observe the strategies (sensitivity parameter values) of its Moore neighborhood, and then it can adjust its own parameters corresponding to the best-response rule (3.19). It was also shown in [25] that impatient agents are able to overtake patient agents in conflict situations.

In the following, a step-by-step description of the simulation model [26] is given. Initially the agents are spread randomly in the evacuating space. At start, no agent plays the game and initial strategy is Patient for all agents. This explanation follows closely the presentation in [26].

1. At the beginning of each time step, the model calculates the estimated evacuation time T_i for each agent i. If $T_i > T_{ASET} - T_0$, agent i plays the game. In the setup explained in Section 4.1, $T_{ASET} - T_0 = 0$, so in the simulations done in this thesis the agents play always the game.

2. Strategies of the agents are updated with shuffle update scheme. Agent i chooses the strategy according to the best-response function (3.19) after observing the strategies of agents in its Moore neighborhood.

3. The CA sensitivity parameters of agent i are updated corresponding to the strategy chosen. The parameter values are chosen as follows:

   (a) Impatient: $k_D = 1.0$ and $k_S = 10.0$.

   (b) Patient: $k_D = 1.0$ and $k_S = 1.0$.

4. The agents move. NB: Moving happens in von Neumann neighborhood even though observing other agents' strategies was done in Moore neighborhood!

5. Go back to 1. The procedure continues until every agent has exited the evacuating space.

Because the sensitivity parameters are already connected straight to the strategies, it is easy to add the model the feature of different agent types. Instead of having global T_0 and T_{ASET} in the model, these parameters are associated to each agent personally. The agents are set to get their agent type randomly out of a probability distribution that can be controlled with a global model input parameter. The type with higher value of T_{ASET} is called *risk-averse type*, and the type with lower value of it is called *risk-taking type*.

Chapter 5

Simulation results

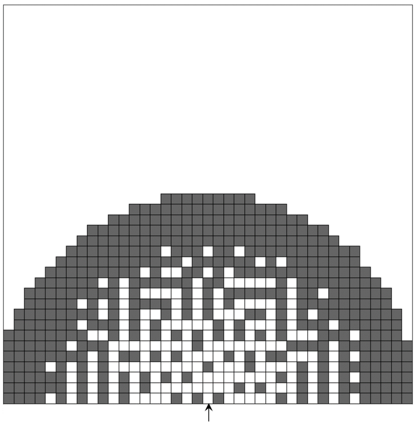
In the simulation model [26], agents are allowed to have different cost functions. Matlab is used as the simulation software. In Section 5.1 static configurations are studied to see which kind of spatial equilibrium patterns are formed in front of the exit. Results from simulations, where agents are able to move, are presented in Section 5.2. The parameter values presented in Figure 4.1 are used as default values for different agent types.

5.1 Static configurations

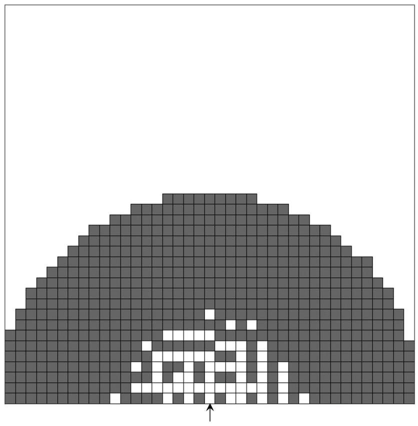
Next, equilibrium configurations resulting from the model with one agent type are compared with the equilibrium configuration of two types of agents. It was shown in [13] that subregions with different proportions of impatient agents are formed in the spatial equilibrium. Each subregion is built out of single category of elementary configuration blocks presented in Figure 3.3. These different areas are always at certain distance round the exit, which is because along arcs of certain exit-centered circles the agents have the same estimated evacuation time.

Figure 5.1 illustrates the results of the simulations. For simulations with two agent types, each agent is given its type randomly with equal probabilities for the both types. Figures 5.1a-5.1b correspond to the results in [13], so the model coded for this thesis can be considered working properly. The structure of Figure 5.1c ended up to show out as expected, namely a mixture of the two previous.

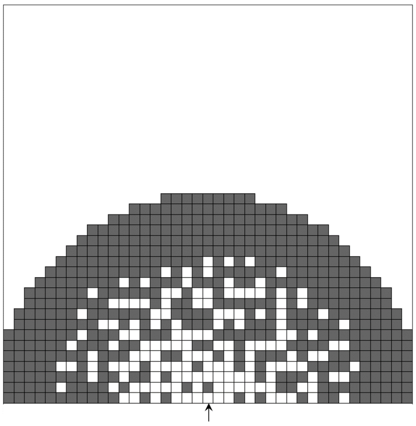
To understand Figure 5.1d, let's look again the game area concept defined



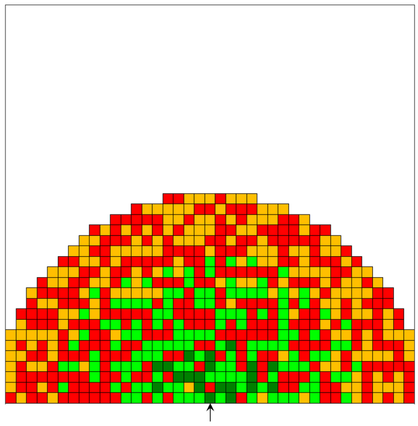
(a) Risk-averse type: $T_{ASET} = 300$.



(b) Risk-taking type: $T_{ASET} = 100$.



(c) Mixed population: 50 % risk-averse and 50 % risk-taking.



(d) Same situation as in (c). Colors reveal the different agent types.

Figure 5.1: Static spatial equilibrium configurations for 628 agents in 39x39 cell grid. In pictures (a)-(c) dark gray squares represent impatient agents and white squares represent patient agents. In picture (d) colors are: orange impatient risk-averse, red impatient risk-taking, light green patient risk-averse, and dark green patient risk-taking.

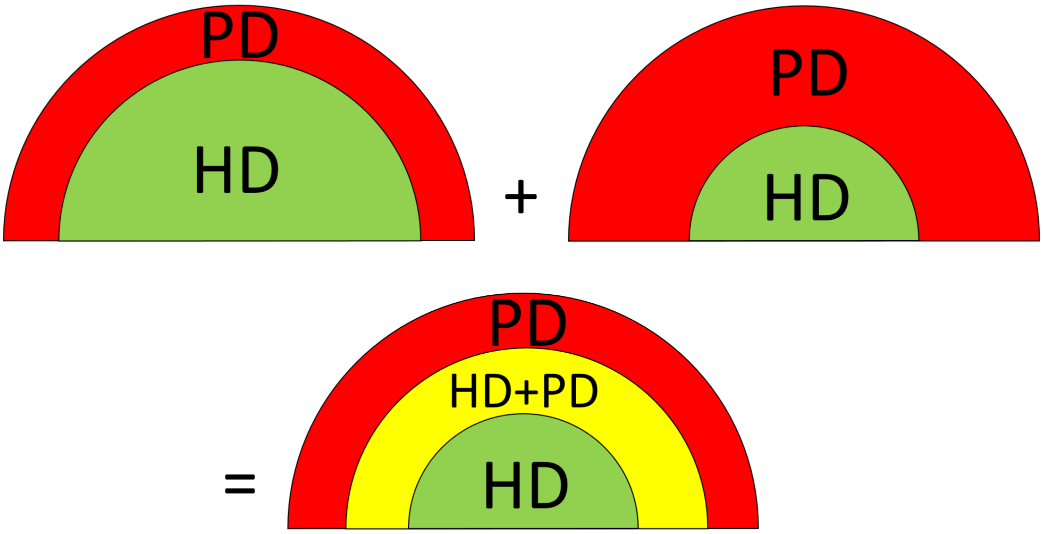


Figure 5.2: Formation of different game areas. In the upper two pictures there are just two game separated by the distance, where $T_i = T_{ASET}$. Merging the two upper pictures results the lower picture, where in the middle area some agents are playing HD while the others play PD.

in Section 3.3. Recall the interpretation of T_{ASET}: after $T_i > T_{ASET}$ the conditions become critical and the agents start to act only Impatient. This border where the game changes from HD to PD can easily be seen both in figures 5.1a and 5.1b. Figure 5.2 explains how the static equilibrium of the two types of agents population is formed. Inside the inner border of the two cases, all agents are playing HD whether they are risk-averse or risk-taking. Same way, outside the outer border all agents are playing PD, i.e. only Impatient. In between these two game areas is a third game area, where risk-averse agents play HD but risk-taking agents play PD.

When distributing different agent types equally in the area, as in our example, a very interesting phenomenon can be seen. Just outside the inner border, all risk-averse agents end up choosing Patient, and this is driven by the fact that all risk-taking agents choose Impatient in the same area. This can be seen in Figure 5.1d as an arc with only light green and red squares. Compare the situation in this location to the same location in Figure 5.1a with only risk-averse agents. At this distance the border $T_{ASET} = 300$ is still so far away that only a few act Impatient. When risk-taking agents, who all act only Impatient, are added to this distance, the situation forces the

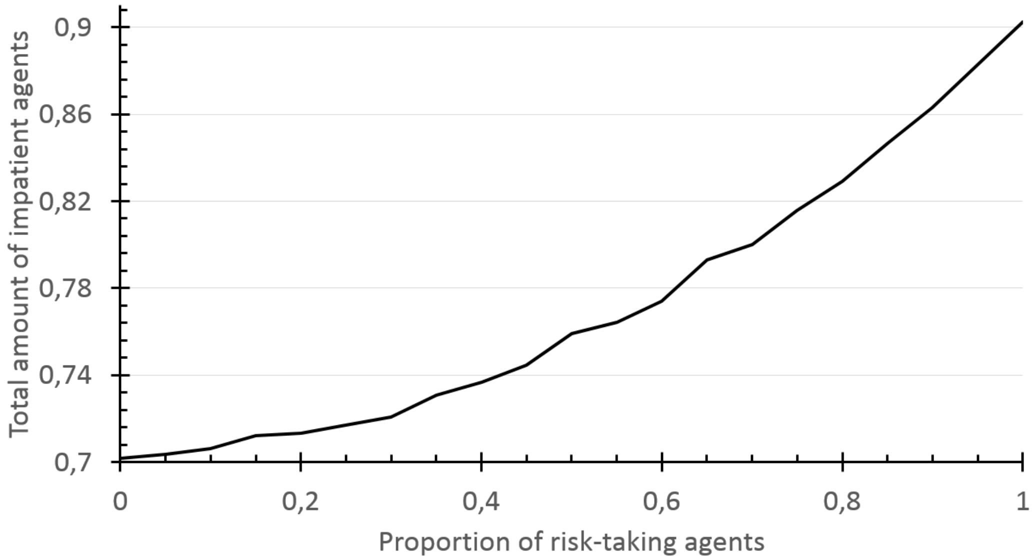


Figure 5.3: Proportion of impatient agents in different static configurations of 628 agents in 39x39 cell grid. The increasing seems quadratic at first, but changes to linear approximately when proportion of risk-taking agents is larger than 0.7.

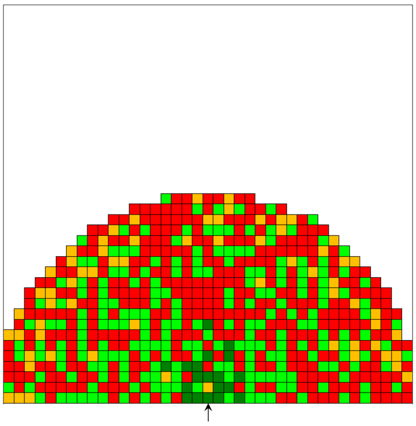
risk-averse agents to avoid acting Impatient. Likely at this distance every risk-averse agent has a risk-taking agent in its Moore neighborhood, who is acting Impatient anyway, so their optimal choice in this situation is acting Patient. The total amount of impatient agents as a function of the proportion of risk-taking agents in the population is plotted in Figure 5.3.

Figure 5.4 presents simulations, in which the parameter T_{ASET} for both agent types is varied. The same effects noted before can be seen here with different locations of the game area borders.

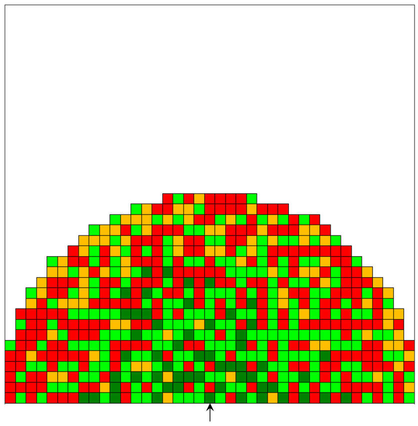
5.2 Cellular automaton with different risk attitudes

In CA simulations, value for friction parameter was chosen to be $\mu = 0.6$. This is large enough to produce faster-is-slower effect described in Section 4.1. For diffuse and decay parameter values, $\alpha = 0.3$ and $\delta = 0.3$ are chosen. Same parameter values were also used in simulations made in [26].

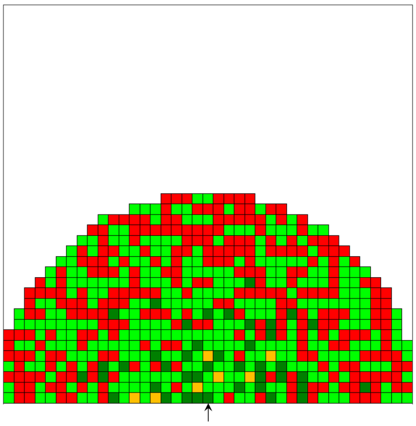
Figure 5.5 shows snapshots of evacuation done with the population presented



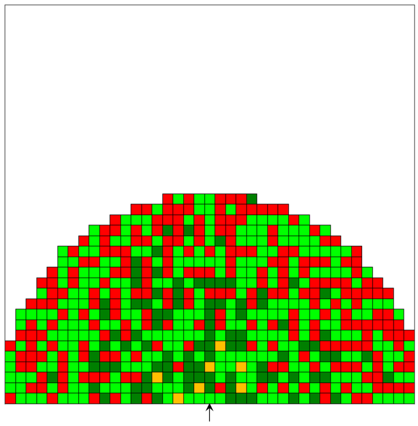
(a) Risk-averse type: $T^1_{ASET} = 500$.
Risk-taking type: $T^2_{ASET} = 100$.



(b) Risk-averse type: $T^1_{ASET} = 500$.
Risk-taking type: $T^2_{ASET} = 300$.



(c) Risk-averse type: $T^1_{ASET} = 5000$.
Risk-taking type: $T^2_{ASET} = 300$.



(d) Risk-averse type: $T^1_{ASET} = 5000$.
Risk-taking type: $T^2_{ASET} = 500$.

Figure 5.4: Further simulations of the static equilibrium configurations with different values of paratemer T_{ASET}. Number of agents, size of the grid, mixing ratio and coloring correspond to the case in Figure 5.1

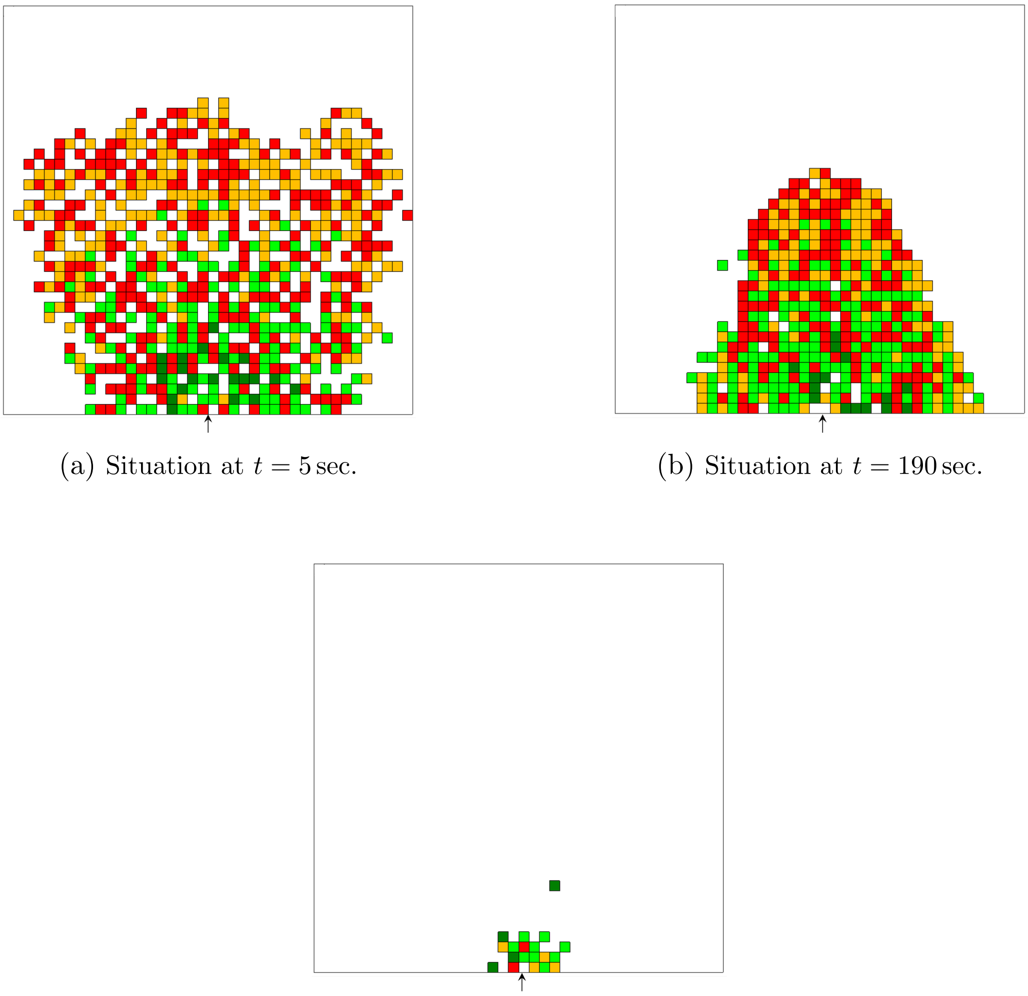
in Figure 5.1d. At first, all 628 agents are placed randomly in 39x39 cell grid, and everyone starts with strategy Patient. At the beginning of each time step, strategies are updated according to best-response rule until an equilibrium is reached. The whole evacuation took approximately 600 seconds, which makes the average exit flow approximately 1 agent / second. As can be seen, pretty quickly agents formation in front of exit starts to resemble a half-circle.

Looking qualitatively at Figure 5.5, it seems that risk-taking agents overtake risk-averse agents in the evacuation situation. At first, ratio of risk-takers in the evacuation space is set to be 50 %. In Figure 5.5b, the ratio has decreased to approximately 40 %. In the very end of the evacuation, Figure 5.5c, only 6/19 are anymore risk-taking agents, which makes the ratio go near 30 %.

CA simulations were also done in a different evacuation space geometry. In Figure 5.6, the crowd is first put to the room on the left-hand side in the pictures, and then put to evacuate through a narrow hallway towards the exit in the room on the right-hand side. Except for the room geometry, parameters were kept the same as in the Figure 5.5 simulation. The hallway fills up with agents pretty quickly, and thus in Figure 5.6b snapshot two half-circle formations can be seen. One half-circle is formed in the first room at the entrance of the hallway, and the other in front of the exit. In the half-circle on the left-hand side, most of the agents are acting Impatient because they are considerably far away from the exit. Also in this case, the risk-taking seem to overtake the risk-averse agents.

But what about the effect of the proportion of risk-taking agents to the total evacuation time of the crowd? This was tested in 20x20 grid with 172 agents. The setups were made smaller to decrease simulation time. Agents were put to half-circle form in front of the exit at start to minimize randomness effect to evacuation time caused by agents wandering around the room in the beginning. Crowds evacuation time was measured for 11 cases, risk-taking agent proportions being from 0 to 1 with interval 0.1. For each proportion 100 simulations were made.

Figure 5.7 shows the average evacuation times for different proportions of risk-taking agents in two setups. The standard deviation in Figure 5.7a cases was approximately 6.4 seconds, and in Figure 5.7b cases approximately 6.7 seconds. These values are relatively large, because the difference of the average evacuation time minimum and maximum is only 9.5 seconds in Figure 5.7a, and 10.6 seconds in Figure 5.7b. The large standard deviation is due the fact that, even though the agents are put at the beginning in a half-circle form, clogging is stochastic phenomenon [13], and thus has a significant effect on different simulation runs. Other factors increasing the stochastic nature of

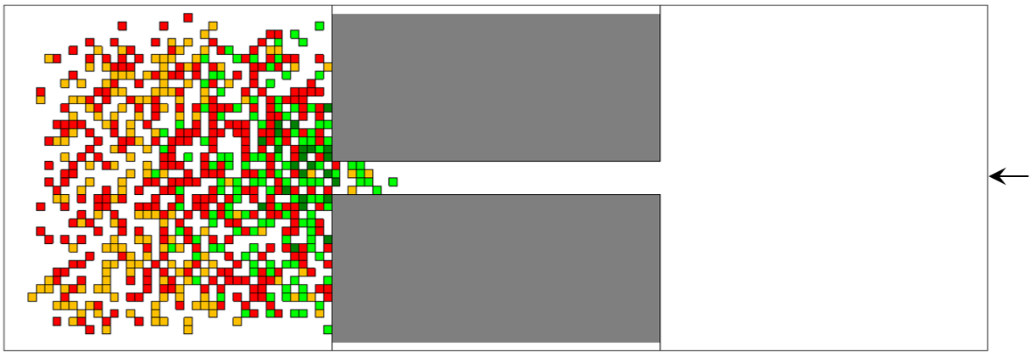


(a) Situation at $t = 5$ sec.

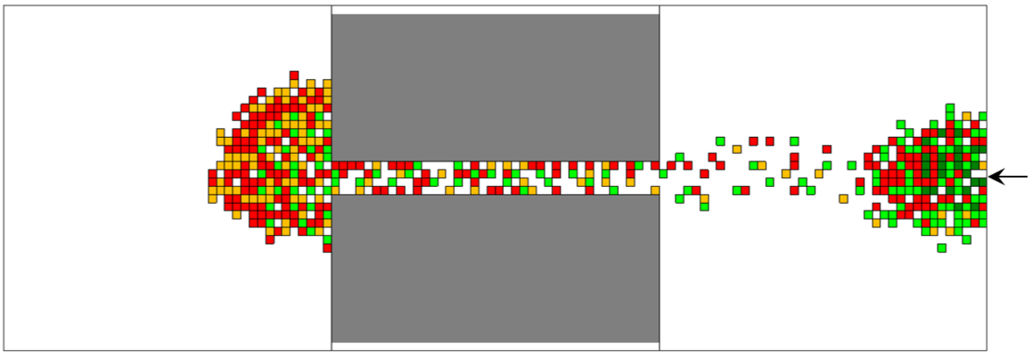
(b) Situation at $t = 190$ sec.

(c) Situation at $t = 575$ sec.

Figure 5.5: Snapshots of CA evacuation. Agent colors correspond to Figure 5.1d. Risk-taking agents seem to overtake risk-averse agents, since the proportion of risk-takers decreases over time among those still not exited from the evacuation space.



(a) Situation at $t = 5$ sec.



(b) Situation at $t = 190$ sec.

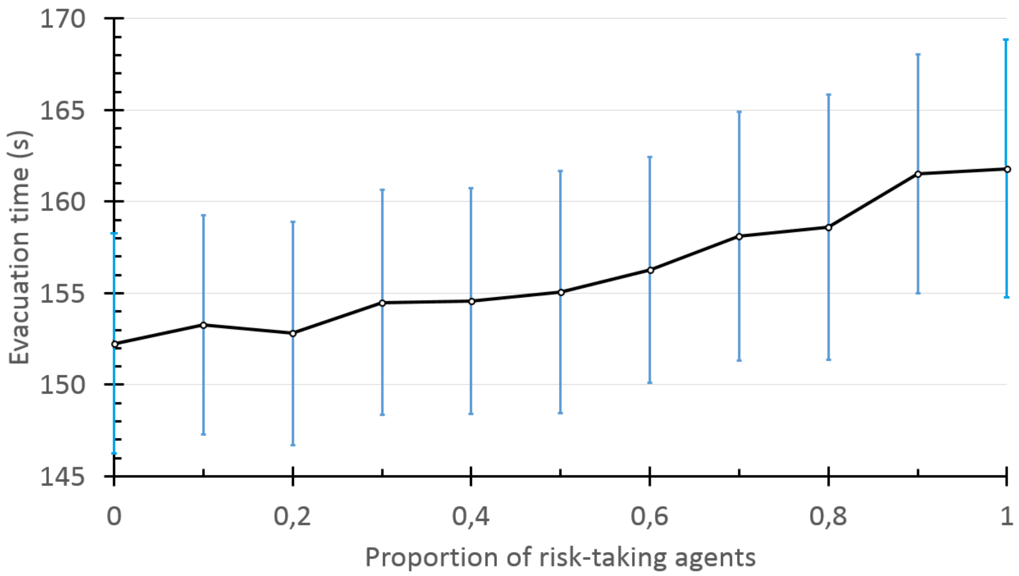
(c) Situation at $t = 575$ sec.

Figure 5.6: Snapshots of CA evacuation in hallway setup. Agent colors correspond to Figure 5.1d.

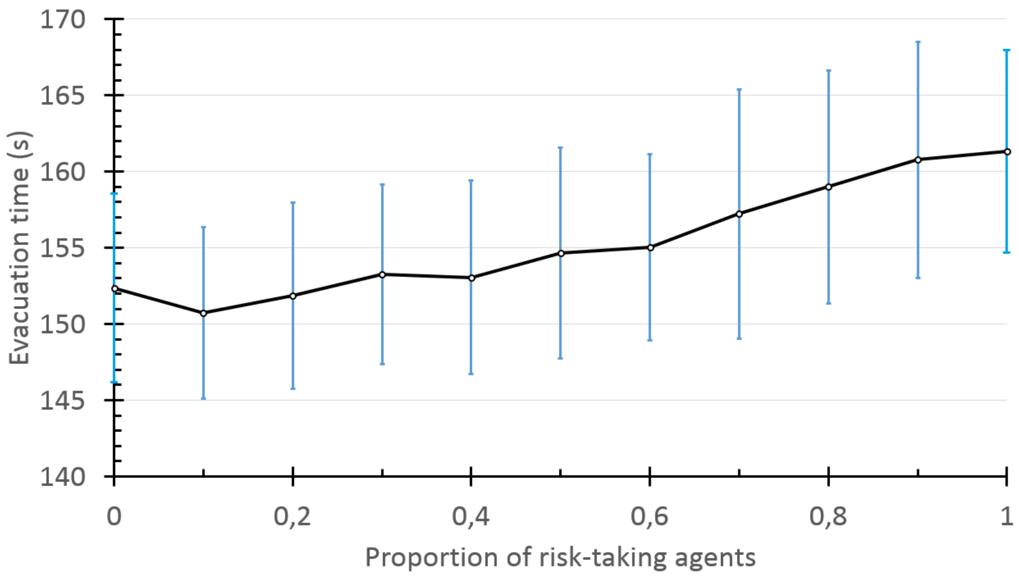
the model are resolving the conflict situations, and the movement in general according to the transition probabilities.

The most interesting phenomenon is the decrease in the average evacuation time, when proportion of risk-taking agents is approximately 0.1-0.2. The curves in Figure 5.7 are mainly rising as expected, so, why seems there to be a local minimum at this point? One hypothesis is that adding a small amount of impatient agents to area occupied mainly out of patient agents could actually make the crowds evacuation *faster*. Consider the pictures in Figure 5.1 again. Most of the agents act Patient near the exit both in Figures 5.1a and 5.1b. These agents near the door don't feel the situation threatening, and thus don't rush to the door, even if there is a big crowd coming from behind.

For really small amount of agents, taking the shortest path, i.e. acting Impatient, is the fastest way to evacuate. Clogging occurs when the number of agents rises: the agents don't get so fast anymore to the door because they end up in conflicts in the middle. On the other hand, near the exit it would be beneficial to the whole crowd to evacuate as fast as possible. When parameter T_{ASET} decreases, the crowd evacuation time increases because the number of impatient agent increases (showed in [13]). Now, when most of agents near the exit are patient and risk-averse, a suitable small amount, 10 - 20 % of the crowd, risk-taking agents are added, and they act more probably Impatient than the risk-averse near the exit. If the hypothesis is right, in this case the crowds evacuation time is lower than if there would be a smaller proportion of risk-taking agents. Again, the proportion of risk-taking agents can't be sufficiently larger than the suitable amount, since then clogging starts to slow down the evacuation.



(a) Risk-averse type: $T_{ASET} = 300$. Risk-taking type: $T_{ASET} = 100$.



(b) Risk-averse type: $T_{ASET} = 500$. Risk-taking type: $T_{ASET} = 100$.

Figure 5.7: Average evacuation time of 172 agents with different proportion of risk-taking agents in the evacuating crowd.

Chapter 6

Discussion

This thesis aimed to study the effect of different risk attitudes in agent-based computational model for evacuation. Both the static strategy configurations and the movement of agents were studied. Cellular automaton was chosen as the computational framework for moving the agents. In CA, agents move in a discrete grid, and also the time advances discretely. In terms of computation time, CA is very efficient.

The term panic is commonly misused to describe a crowd that evacuates in a disastrous way. However, research has shown that even stampedes that caused deaths occurred under conditions where evacuees behave rationally. Evacuees' decision making can be modeled with spatial egress game where the agents have two strategies to choose from: Impatient and Patient. Depending on the distance to the exit, this game is either Hawk-Dove or prisoner's dilemma. It can be shown that, with this behavior, individuals trying their best to evacuate as fast as possible can cause the slowing down of the whole crowds evacuation.

Spatial game for egress congestion is a decision-making module for agents that has been successfully coupled with CA, and also for continuous time and space evacuation software FDS+Evac. In the model of this thesis, spatial cellular automaton game was added a feature to enable multiple cost functions for the agents. The strategy choices, Impatient or Patient, were the same as before, but the agents have different tendencies to choose among the strategy alternatives, and these tendencies are based on their personal cost function.

The different risk attitudes studied in this thesis were limited to two alternatives: risk-averse type and risk-taking type. Other agents do not observe the type of the other agents, but only the strategies they play. A risk-taking agent is keener to play Impatient than a risk-averse agent. The different agent types, and the corresponding cost functions, reflect a state-of-mind and personal risk attitude, which also in real life has major impact in decision making.

The model in this thesis showed two interesting phenomenon caused by different agent types. First was the formation of three game areas. In a normal case of homogenous agents only two game areas exist: HD area near the exit and PD area outside the border, where estimated evacuation time is longer than T_{ASET}. Because the different agent types have different values for T_{ASET}, between the borders comes a third area where one type is playing HD (risk-averse) and the other PD (risk-taking). In this third area the risk-taking agents in a way "reserve" all the places to play Impatient forcing the risk-averse agents to play more Patient than normally.

The other noticed effect was the tiny decrease in the whole crowds evacuation times, when sufficiently small amount of risk-taking agents were mixed with the risk-averse agents. Reason for this could be that near the exit the evacuating as fast as possible would be beneficial for the whole crowd, but playing Patient is there really common because estimated evacuation time is much shorter than T_{ASET} and playing Impatient is feared to cause clogging. This hypothesis would need more studying, because the standard deviations around the whole crowds estimated evacuation times were sufficiently large compared to differences in the evacuation times between the different proportions of risk-taking agents in the population.

For other future research ideas, the effect of risk attitude should be studied also combined to other computational evacuation models than CA. For example, FDS+Evac would be an excellent candidate, because it already has spatial egress game in it. Although CA and FDS+Evac give similar results for modeling an evacuating crowd, the functioning logic is completely different. Impatient behavior in FDS+Evac is shown as higher desired walking speed and the agents push each other when trying to get out from the evacuation area. It would be interesting to see how multiple cost functions would affect that behavior.

Last idea is to make more experiments on the cost function. How would the situation change if even more agent types were added? Or moreover, what if agent type were a continuous characteristic? Or dynamically changing in time? Changing both T_{ASET} and T_0 without strict connection to each other could also reveal effects not found in this thesis. Finally, the shape of the cost function could also be altered, and see if the alternations have a real-life interpretation.

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Appendix: Prisoner's dilemma and Nash equilibrium

Merrill Flood and Melvin Dresher developed originally a model for two player game of cooperation and conflict in 1950. Albert W. Tucker gave this game the following, nowadays famous, formalization[21]:

"Two members of a criminal gang are arrested and imprisoned. Each prisoner is in solitary confinement with no means of speaking to or exchanging messages with the other. The prosecutors do not have enough evidence to convict the pair on the principal charge. They hope to get both sentenced to a year in prison on a lesser charge. Simultaneously, the prosecutors offer each prisoner a bargain. Each prisoner is given the opportunity either to: betray the other by testifying that the other committed the crime, or to cooperate with the other by remaining silent. Here is the offer:

- If A and B each betray the other, each of them serves 2 years in prison
- If A betrays B but B remains silent, A will be set free and B will serve 3 years in prison (and vice versa)
- If A and B both remain silent, both of them will only serve 1 year in prison (on the lesser charge)"

Hence Tucker's interpretation, the game got the name prisoner's dilemma (PD). The game can be presented in 2×2 matrix form:

		Player B	
		Cooperate (C)	Defect (D)
Player A	Cooperate (C)	$1, 1$	$3, 0$
	Defect (D)	$0, 3$	$2, 2$

,

where players try to minimize the prison time. The rational outcome of this game is that both players choose **D** as their strategy. Why? If both chose C as their strategy, the criminals would minimize their joint time in prison. However, if player A knows that player B will certainly play C, then player A by playing D would not end up in prison at all. Now, player B thinks the situation in symmetrical way, and thus the criminals end up serving the maximum joint time in prison.

The strategy pair (D,D) is the (pure strategy) *Nash equilibrium* (NE) of PD. More generally, in NE a player cannot gain a better outcome by changing her own strategy if the other hold on to their NE strategies. See the case for PD: if for example player A changed from D to C, while player B holds on playing D, then player A would only worsen her own situation by the change. Strategy pair (C,C) is not NE, since if one player would change, and the other player would hold on, the one who changed would benefit from the change.

Furthermore, any 2×2 game

		Player B: C	Player B: D
Player A	C	R, R	S, T
	D	T, S	P, P

is a PD if $T > R > P > S$. The letters come from words *temptation*, *reward*, *punishment* and *sucker*. By cooperating the players would earn the reward, but succumbing to the temptation (and not wanting to be a sucker) the players end up suffering the punishment. If $T > R > S > P$, the game is HD. There are two (pure strategy) Nash equilibria in HD, namely (C,D) and (D,C). Also, there exist a mixed strategy NE in HD with the probability from Equation (3.8). In EGT terms, Cooperate strategy correspond to Dove strategy, and Defect correspond to Hawk.

NE has many similarities to ESS described in Section 3.1. However, ESS is a stronger concept than NE. As stated before, playing any pure strategy in HD is not optimal for a population of players despite the game has two pure strategy Nash equilibia. Yet, all 2×2 games have at least one ESS and in the case of HD it is the mixed strategy NE. For PD, the pure strategy NE (D,D) is also the ESS of the game.